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Recalculating Gravity: A Correction of Bergstrand's 1985 Frictionless Case

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LINK TO ABSTRACT

Jeffrey Bergstrand (1985) derives a theoretical gravity equation for bilateral international trade flows focusing on the role of prices. The gravity equation has served as the basis for thousands of theoretical and empirical works in the field of international economics.

Bergstrand develops a general equilibrium model of world trade, which is based on utility and profit maximization in N countries endowed with one production factor each. He uses nested utility and production functions based on constant elasticity of substitution (CES) and constant elasticity of transformation (CET), respectively. The model generates N^2 partial equilibrium subsystems of 4 equations each with 4 endogenous variables and $3N$ constraints. This system of $4N^2 + 3N$ equations results in the general gravity model if the small market assumption—the neglectable impact of the market between country i and j on other markets—and the assumptions of identical preferences and technologies across countries hold.

1. Paderborn University, 33098 Paderborn, Germany. I would like to express my special thanks to Jeffrey H. Bergstrand and Bernard M. Gilroy for their helpful comments and suggestions.

$$PX_{ij} = \underbrace{Y_i^{\frac{\sigma-1}{\gamma+\sigma}} Y_j^{\frac{\gamma+1}{\gamma+\sigma}}}_{I} \underbrace{C_{ij}^{-\frac{\sigma(\gamma+1)}{\gamma+\sigma}} T_{ij}^{-\frac{\sigma(\gamma+1)}{\gamma+\sigma}} E_{ij}^{\frac{\sigma(\gamma+1)}{\gamma+\sigma}}}_{II}$$

$$\cdot \underbrace{\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{-\frac{(\sigma-1)(\gamma-\eta)}{(1+\gamma)(\gamma+\sigma)}}}_{III} \underbrace{\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\right)^{\frac{(\gamma+1)(\sigma-\mu)}{(1-\sigma)(\gamma+\sigma)}}}_{IV}$$

$$\cdot \underbrace{\left[\left(\sum_{k=1,k\neq i}^{N} P_{ik}^{1+\gamma}\right)^{\frac{1+\eta}{1+\gamma}} + P_{ii}^{1+\eta}\right]^{-\frac{\sigma-1}{\gamma+\sigma}}}_{V} \underbrace{\left[\left(\sum_{k=1,k\neq j}^{N} P_{kj}^{1-\sigma}\right)^{\frac{1-\mu}{1-\sigma}} + P_{jj}^{1-\mu}\right]^{-\frac{\gamma+1}{\gamma+\sigma}}}_{VI}$$

This equation is the same as equation (14) in Bergstrand's original paper (Bergstrand 1985, 477), divided into VI parts. The value of bilateral trade is dependent on both countries' gross domestic products Y, the gross transport factor C, the tariff rate T, the exchange rate E, the f.o.b. price of i's good in k P_{ik}, the c.i.f. price of k in j P_{kj}, and domestic prices and elasticities, where $\sigma(\mu)$ is the elasticity of substitution in consumption between imported goods (between imported and domestic goods) and $\gamma(\eta)$ is the elasticity of transformation in production between export markets (between foreign and domestic markets). This equation is often cited as Bergstrand's generalized gravity model. It is used to define sufficient control variables for empirical trade analysis and to provide theoretical justification for gravity.

To get a frictionless gravity model that excludes all price terms, further assumptions have to be made. Assuming perfect substitutability, perfect commodity arbitrage, zero tariffs, zero transport costs and normalizing exchange rate to unity implies that $C_{ij}=T_{ij}=E_{ij}=1$, $P_{ik}=P_{kj}=P_{ii}=P_{jj}=\overline{P}$ for all i, j, k and $\sigma=\mu=\gamma=\eta=\infty$. Bergstrand's simplification resulted in his equation (15), $PX_{ij}=(1/2)Y_i^{1/2}Y_j^{1/2}$—but recalculating the derivation, as shown in the following, actually leads to $PX_{ij}=Y_i^{1/2}Y_j^{1/2}$. The coefficient (1/2) in Bergstrand's equation (15) should have been omitted. This correction has been confirmed by Professor Bergstrand as part of the valuable comments he provided on this paper.

$$I: Y_i^{\frac{\sigma-1}{\gamma+\sigma}} Y_j^{\frac{\gamma+1}{\gamma+\sigma}} = Y_i^{\frac{1}{2}} Y_j^{\frac{1}{2}}$$

$$II: C_{ij}^{-\frac{\sigma(\gamma+1)}{\gamma+\sigma}} T_{ij}^{-\frac{\sigma(\gamma+1)}{\gamma+\sigma}} E_{ij}^{\frac{\sigma(\gamma+1)}{\gamma+\sigma}} = 1$$

$$III: \left(\sum_{k=1,k\neq i}^{N} \bar{P}^{1+\gamma}\right)^{-\frac{(\sigma-1)(\gamma-\eta)}{(1+\gamma)(\gamma+\sigma)}} = \left(\sum_{k=1,k\neq i}^{N} \bar{P}^{1+\gamma}\right)^{-\frac{(\sigma-1)(0)}{(1+\gamma)(\gamma+\sigma)}} = \left(\sum_{k=1,k\neq i}^{N} \bar{P}^{1+\gamma}\right)^{0} = 1$$

$$IV: \left(\sum_{k=1,k\neq j}^{N} \bar{P}^{1-\sigma}\right)^{\frac{(\gamma+1)(\sigma-\mu)}{(1-\sigma)(\gamma+\sigma)}} = \left(\sum_{k=1,k\neq j}^{N} \bar{P}^{1-\sigma}\right)^{\frac{(\gamma+1)(0)}{(1-\sigma)(\gamma+\sigma)}} = \left(\sum_{k=1,k\neq j}^{N} \bar{P}^{1-\sigma}\right)^{0} = 1$$

$$V: \left[\left(\sum_{k=1,k\neq i}^{N} \bar{P}^{1+\gamma}\right)^{\frac{1+\eta}{1+\gamma}} + \bar{P}^{1+\eta}\right]^{-\frac{\sigma-1}{\gamma+\sigma}} = \left\{\left[\left((N-1)\bar{P}^{1+\gamma}\right)^{\frac{1}{1+\gamma}}\right]^{1+\eta} + \bar{P}^{1+\eta}\right\}^{-\frac{\sigma-1}{\gamma+\sigma}}$$

$$= \left\{\left[(N-1)^{\frac{1}{1+\gamma}}\bar{P}\right]^{1+\eta} + \bar{P}^{1+\eta}\right\}^{-\frac{\sigma-1}{\gamma+\sigma}} = \left\{(N-1)^{\frac{1+\eta}{1+\gamma}}\bar{P}^{1+\eta} + \bar{P}^{1+\eta}\right\}^{-\frac{\sigma-1}{\gamma+\sigma}}$$

$$= \{N\bar{P}^{1+\eta}\}^{-\frac{\sigma-1}{\gamma+\sigma}} = \left\{N^{\frac{1}{1+\eta}}\bar{P}\right\}^{-\frac{(\sigma-1)(1+\eta)}{\gamma+\sigma}} = \bar{P}^{-\frac{(\frac{\sigma}{\sigma}-\frac{1}{\sigma})(\frac{1}{\sigma}+\frac{\eta}{\sigma})}{\frac{\gamma}{\sigma}+\frac{\sigma}{\sigma}}} = \bar{P}^{-\frac{1}{2}}$$

$$VI: \left[\left(\sum_{k=1,k\neq j}^{N} \bar{P}^{1-\sigma}\right)^{\frac{1-\mu}{1-\sigma}} + \bar{P}^{1-\mu}\right]^{-\frac{\gamma+1}{\gamma+\sigma}} = \left\{\left[\left((N-1)\bar{P}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}\right]^{1-\mu} + \bar{P}^{1-\mu}\right\}^{-\frac{\gamma+1}{\gamma+\sigma}}$$

$$= \left\{\left[(N-1)^{\frac{1}{1-\sigma}}\bar{P}\right]^{1-\mu} + \bar{P}^{1-\mu}\right\}^{-\frac{\gamma+1}{\gamma+\sigma}} = \left\{(N-1)^{\frac{1-\mu}{1-\sigma}}\bar{P}^{1-\mu} + \bar{P}^{1-\mu}\right\}^{-\frac{\gamma+1}{\gamma+\sigma}}$$

$$= \{N\bar{P}^{1-\mu}\}^{-\frac{\gamma+1}{\gamma+\sigma}} = \left\{N^{\frac{1}{1-\mu}}\bar{P}\right\}^{-\frac{(\gamma+1)(1-\mu)}{\gamma+\sigma}} = \bar{P}^{-\frac{(\frac{\gamma}{\gamma}+\frac{1}{\gamma})(\frac{1}{\gamma}-\frac{\mu}{\gamma})}{\frac{\gamma}{\gamma}+\frac{\sigma}{\gamma}}} = \bar{P}^{\frac{1}{2}}$$

$$\bar{P}X_{ij} = Y_i^{\frac{1}{2}} Y_j^{\frac{1}{2}} \bar{P}^{\frac{1}{2}-\frac{1}{2}} = Y_i^{\frac{1}{2}} Y_j^{\frac{1}{2}}$$

While the empirical implications of this correction may not be serious, there is a formal theoretical relevance. Besides presenting the correct solution of the frictionless case, this note further contributes to the literature on gravity models in three ways.

Bergstrand (1985) introduces the theoretical section of his paper by stating that the gravity equation then in use, $PX_{ij} = \beta_0 Y_i^{\beta_1} Y_j^{\beta_2} D_{ij}^{\beta_3} A_{ij}^{\beta_4} u_{ij}$, lacks a sufficient theoretical foundation. He then defines his goal to be the generation of a gravity equation as similar as possible to this representation. The first equation he derives is from the general equilibrium model. With further constraints, he achieves his goal of creating a frictionless case closer to the widely used equation. With the correction of the constant in this special case, we get even closer to his goal.

Not only the general equilibrium model is frequently used in empirical studies. Some authors also cite the frictionless case as a base for further theoretical studies (e.g., Földvári 2006, 48; Ramesh 2017, 149). These works also need to be corrected.

The range of theoretical derivations of gravity models has grown significantly over time, with the most cited models next to Bergstrand being Anderson 1979; Anderson and van Wincoop 2003; Deardorff 1998; and Eaton and Kortum 2002. Meta-papers (such as Head and Mayer 2014), which have set themselves the task of classifying and comparing gravity approaches, occasionally criticize a lack of comparability. Corrections such as the one presented here contribute to such comparability.

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