



Response to “Comment on Measuring the Size of the Shadow Economy Using a Dynamic General Equilibrium Model with Trends”

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[LINK TO ABSTRACT](#)

In Solis-Garcia and Xie (2018), our aim was to propose a working method for measuring the size and properties of the shadow economy based on a dynamic deterministic general equilibrium (DGE) model. The approach relied on the dynamics of observed trends along the balanced growth path to account for both the size and the cyclical nature of the shadow economy, as the trends impose a set of equilibrium restrictions over the growth rates of the model variables—including shadow sector output.

In a recent contribution, Manuel Gómez and Adrián Ríos-Blanco (2022) argue that the method proposed by our paper fails to accomplish its goal. In particular, their argument is twofold: First, the assumption of a balanced growth path implies a constant shadow-to-formal output ratio. Second, an error on our end hides this inconsistency and gives rise to an unlucky coincidence, in that it generated results that looked sensible but concealed the original flaw in the method. We acknowledge the mathematical error and concede that using the properties of the balanced growth path indeed results in the consequences they

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point out. We sincerely regret this error.

That said, we believe that the claim of Gómez and Ríos-Blanco (2022) about our paper failing to accomplish its goals to measure the shadow economy is unfounded. More precisely, we can attain these goals when we relax the balanced growth path assumption; this change allows us to obtain precise estimates of the size of the shadow economy using minimal data, even compared to Solis-Garcia and Xie (2018). Our revised procedure retains the original model but streamlines the calculations, dropping the balanced growth path assumption altogether and relying instead on a subset of the equilibrium conditions to pin down the dynamics of the informal sector. Put differently, the procedure below achieves the goals we were set to deliver in our earlier contribution.

Revisiting Solis-Garcia and Xie (2018)

Again, we still believe that the idea underlying the paper is sound. In a nutshell, the restrictions imposed by a DGE model require shadow output to behave in a certain way—it cannot move around in a random way.

Moreover, if shadow sector producers want to remain underground—which, by definition, is what the informal producers desire!—then relying on theory to back out what by definition doesn't want to be measured is a good idea.

In this sense, our DGE model and approach with trend still delivers what it was set to do: derive precise estimates of the size of the shadow economy using minimal data. The only update we made to our procedure is to drop the balanced growth path assumption. This means our updated method is more general than the original version and calls for less restrictions. We now describe how the updated procedure is able to pin down the dynamics of the informal sector using a subset of original equilibrium conditions.

Equilibrium conditions

The model remains the same but for a minor modification: We set the hours trend Γ_{Ht} to unity. Note that this is without loss of generality, as we still allow shadow- and formal-sector hours to trend up or down. In this sense, the set of equilibrium conditions is virtually identical, and is reproduced below for convenience:

$$C_t + \Gamma_{At} X_t + G_t = K_t^\alpha (\Gamma_{Ft} N_{Ft})^{1-\alpha} + (\Gamma_{St} N_{St})^\eta \quad (1)$$

$$K_{t+1} = (1 - \delta)K_t + X_t \quad (2)$$

$$N_t = N_{Ft} + N_{St} \quad (3)$$

$$\Gamma_{At} C_t^{-\sigma} = \alpha \beta C_{t+1}^{-\sigma} (1 - \tau_{t+1}) K_{t+1}^{\alpha-1} (\Gamma_{F,t+1} N_{F,t+1})^{1-\alpha} + \beta (1 - \delta) \Gamma_{A,t+1} C_{t+1}^{-\sigma} \quad (4)$$

$$\phi N_t^{\chi} = (1 - \alpha) C_t^{-\sigma} (1 - \tau_t) K_t^{\alpha} \Gamma_{Ft}^{1-\alpha} N_{Ft}^{-\alpha} \quad (5)$$

$$\eta (1 - \rho \hat{s} \tau_t) \Gamma_{St}^{\eta} N_{St}^{\eta-1} = (1 - \alpha) (1 - \tau_t) K_t^{\alpha} \Gamma_{Ft}^{1-\alpha} N_{Ft}^{-\alpha} \quad (6)$$

$$G_t = \tau_t K_t^{\alpha} (\Gamma_{Ft} N_{Ft})^{1-\alpha} + \rho \hat{s} \tau_t (\Gamma_{St} N_{St})^{\eta} \quad (7)$$

$$Y_{Ft} = K_t^{\alpha} (\Gamma_{Ft} N_{Ft})^{1-\alpha} \quad (8)$$

$$Y_{St} = (\Gamma_{St} N_{St})^{\eta} \quad (9)$$

$$Y_t = Y_{Ft} + Y_{St} \quad (10)$$

$$P_{Xt} = \Gamma_{At} \quad (11)$$

Note that our choice to set $\Gamma_{Ht} = 1$ only affects condition (5). Also, and consistent with the parametrization choice in Solis-Garcia and Xie (2018), we will set the probability of an audit (ρ) to zero, which modifies (6) and (7) slightly.

Equilibrium growth rates

Gómez and Ríos-Blanco (2022) show that our original choice to use the balanced growth path conditions (namely, $g_{Y_S} = g_{Y_F}$) results in a constant shadow-to-formal output ratio. Our turnaround is to discard the assumption of balanced growth and focus on period-by-period growth rates instead; as we show below, we have enough observed growth rates to back out the size and dynamics of the shadow economy—though we still need a base year value $Y_{[S/F], t_0}$.

To make this operational, note that conditions (5), (6), and (9) imply

$$g_{Nt}^{\chi} = g_{Ct}^{-\sigma} g_{Y_{Ft}} g_{N_{Ft}}^{-1} \quad (12)$$

$$g_{Y_{St}} g_{N_{St}}^{-1} = g_{Y_{Ft}} g_{N_{Ft}}^{-1} \quad (13)$$

$$g_{Y_{St}} = \left(g_{St} g_{N_{St}} \right)^\eta. \quad (14)$$

To reiterate, equations (12)–(14) are not assumed to hold along the balanced growth path, as the time subscripts suggest.

From (12), we get a measure for the total labor growth rate (g_{N_t}) as a function of the growth rates of consumption (g_{C_t}) and formal output and hours ($g_{Y_{Ft}}$ and $g_{N_{Ft}}$, respectively):

$$g_{N_t} = \left(g_{C_t}^{-\sigma} g_{Y_{Ft}} g_{N_{Ft}}^{-1} \right)^{1/\chi}. \quad (15)$$

Similarly, substituting (14) into (13) gives

$$\left(g_{St} g_{N_{St}} \right)^\eta g_{N_{St}}^{-1} = g_{Y_{Ft}} g_{N_{Ft}}^{-1},$$

which allows us to express the shadow productivity growth rate (g_{St}) as a function of shadow and formal labor growth rates ($g_{N_{St}}$ and $g_{N_{Ft}}$, respectively), the growth rate of formal output ($g_{Y_{Ft}}$), and parameter η . A bit of algebra results in

$$g_{St} = \left(g_{Y_{Ft}} g_{N_{Ft}}^{-1} g_{N_{St}}^{1-\eta} \right)^{1/\eta}. \quad (16)$$

Equations (15) and (16) are all we need to back out the dynamic properties of the shadow economy.

Uncovering the shadow economy

Finding shadow labor for the base period

By construction, the shadow-to-formal output ratio in base year t_0 equals

$$Y_{[S/F],t_0} \equiv \frac{Y_{S,t_0}}{Y_{F,t_0}}. \quad (17)$$

Substituting the expression for shadow output, equation (9), we get

$$Y_{[S/F],t_0} = \frac{\left(\Gamma_{S,t_0} N_{S,t_0}\right)^\eta}{Y_{F,t_0}}.$$

From here we can solve for shadow labor in the base period

$$N_{S,t_0} = \frac{\left(Y_{[S/F],t_0} Y_{F,t_0}\right)^{1/\eta}}{\Gamma_{S,t_0}}$$

and, without loss of generality, note we can normalize Γ_{S,t_0} to unity. Thus, base-period shadow labor is simply

$$N_{S,t_0} = \left(Y_{[S/F],t_0} Y_{F,t_0}\right)^{1/\eta}. \quad (18)$$

By inspection, all the terms in the right-hand side of the expression above are observable, except for the value of η . We now explain how to identify this parameter.

Pinning down the value of η

We use the equilibrium condition (6) to solve for η ; as condition (6) holds for every period, it should hold for t_0 as well. Thus,

$$\eta = \frac{(1-\alpha)\left(1-\tau_{t_0}\right) K_{t_0}^\alpha \Gamma_{F,t_0}^{1-\alpha} N_{F,t_0}^{-\alpha}}{\Gamma_{S,t_0}^\eta N_{S,t_0}^{\eta-1}} = \frac{(1-\alpha)\left(1-\tau_{t_0}\right) Y_{F,t_0} N_{F,t_0}^{-1}}{Y_{S,t_0} N_{S,t_0}^{-1}},$$

where the second equality uses the definitions of formal and shadow output, conditions (8) and (9). Rearranging the negative exponents gives

$$\eta = \frac{(1 - \alpha)(1 - \tau_{t_0})Y_{F,t_0}N_{S,t_0}}{Y_{S,t_0}N_{F,t_0}}.$$

Now, equation (17) implies that $Y_{S,t_0} = Y_{[S/F],t_0} Y_{F,t_0}$; using this expression in the denominator of the expression above—and (18) in the numerator—we get

$$\eta = \frac{(1 - \alpha)(1 - \tau_{t_0})Y_{F,t_0} \left(Y_{[S/F],t_0} Y_{F,t_0} \right)^{1/\eta}}{Y_{[S/F],t_0} Y_{F,t_0} N_{F,t_0}}$$

or, simplifying the numerator,

$$\eta = \frac{(1 - \alpha)(1 - \tau_{t_0})Y_{[S/F],t_0}^{(1-\eta)/\eta} Y_{F,t_0}^{1/\eta}}{N_{F,t_0}}. \tag{19}$$

Note that equation (19) is one equation in η . As in the original paper, we use a fixed-point algorithm to find the value of η that solves the equation:

Algorithm 1 (Fixed point calculation of η)

1. Pick a base year t_0 and obtain data for the tax rate τ_{t_0} , the shadow-to-formal output ratio $Y_{[S/F],t_0}$, formal output Y_{F,t_0} , and formal labor N_{F,t_0} .³
2. Pick values for $\{\alpha, \sigma, \chi\}$. Set a value $M \gg 0$ and build a grid with M elements over the interval $[\eta_L, \eta_H]$, where $\eta_L \geq 1$ (see Remark 5.4 in Solis-Garcia and Xie 2018); call this set N .

3. From condition (7) and our assumption of period-by-period government budget balance, we get

$$\tau_{t_0} = \frac{G_{t_0}}{K_{t_0}^\alpha (\Gamma_{F,t_0} N_{F,t_0})^{1-\alpha}} = \frac{G_{t_0}}{Y_{F,t_0}}.$$

(Note that we have already imposed $\rho = 0$.) These terms can be easily backed out from real-world data (recall that G_t is endogenous in our model).

3. For every $\eta_m \in N$, calculate η following equation (19).
4. Find the entry in N where $|\eta_m - \eta|$ is minimized; call this value η^* .

Given η^* , our calculated value of shadow labor in period t_0 (hereafter, N_{S,t_0}^*) follows equation (18):

$$N_{S,t_0}^* = \left(Y_{[S/F],t_0} Y_{F,t_0} \right)^{1/\eta^*}.$$

Finding total and shadow labor for the full sample

From condition (3), it's clear that total labor for the base year is

$$N_{t_0} = N_{F,t_0} + N_{S,t_0}.$$

Given N_{S,t_0}^* and the observed value for N_{F,t_0} , this value is easily obtained. Now recall equation (15):

$$g_{Nt} = \left(g_{Ct}^{-\sigma} g_{Y_{Ft}} g_{N_{Ft}}^{-1} \right)^{1/\chi}.$$

All the growth rates on the right-hand side of the expression are observable; from here we can calculate the trend Γ_{Nt}^* (normalized so that $\Gamma_{N,t_0}^* = 1$) and then obtain the full series of total labor, $\{N_t^*\}$, as

$$N_t^* = \Gamma_{Nt}^* N_{t_0}^*.$$

Of course, we can use condition (3) again to obtain the values for shadow labor, $\{N_{St}^*\}$, using that

$$N_{St}^* = N_t^* - N_{Ft}.$$

Finding shadow productivity and output for the full sample

Recall equation (16):

$$g_{St} = \left(g_{Y_{Ft}} g_{N_{Ft}}^{-1} g_{N_{St}}^{1-\eta} \right)^{1/\eta}.$$

Given the set of shadow labor values $\{N_{St}^*\}$, all the values in the right-hand side of the equation are available to us. This allows us to calculate the sequence of shadow productivity rates $\{g_{St}^*\}$ and, in a second step, the trend $\{\Gamma_{St}^*\}$ (normalized so that $\Gamma_{S,t_0}^* = 1$). With these values at hand, we obtain the full series of shadow output $\{Y_{St}^*\}$ following the definition of shadow sector output, equation (9):

$$Y_{St}^* = \left(\Gamma_{St}^* N_{St}^* \right)^\eta.$$

Concluding remarks

In their comment, Gómez and Ríos-Blanco (2022, abs.) argue that “the [Solis-Garcia and Xie 2018] method does not serve its purpose because once correctly implemented it generates a constant ratio of shadow to formal production, which is equal to a base-year estimate taken from an external source.”

In this response, we prove that the authors’ assessment is unfounded. While we agree that our 2018 paper is affected by the issues detailed in their comment, the purpose of our approach—to provide a measure of the shadow output—remains intact and is still based on the cross-equation restrictions of a DGE model. Our updated procedure is an improvement of the 2018 given the same model and trend, but asks for less restrictions.⁴

References

- Gómez, Manuel A., and Adrián Ríos-Blanco.** 2022. Comment on Measuring the Size of the Shadow Economy Using a Dynamic General Equilibrium Model with Trends. *Econ Journal Watch* 19(1): 124–132. [Link](#)
- Solis-Garcia, Mario, and Yingtong Xie.** 2018. Measuring the Size of the Shadow Economy Using a Dynamic General Equilibrium Model with Trends. *Journal of Macroeconomics* 56(1): 258–275.

4. A detailed version of this response will be available shortly at the corresponding author’s website.

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[Go to archive of Comments section](#)

[Go to March 2022 issue](#)



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